

Persistent bright solitons in sign-indefinite coupled nonlinear Schrödinger equations with a time-dependent harmonic trap

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Abstract

We introduce a model based on a system of coupled nonlinear Schrödinger (NLS) equations with opposite signs in front of the kinetic and gradient terms in the two equations. It also includes time-dependent nonlinearity coefficients and a parabolic expulsive potential. By means of a gauge transformation, we demonstrate that, with a special choice of the time dependence of the trap, the system gives rise to persistent solitons. Exact single- and two-soliton analytical solutions and their stability are corroborated by numerical simulations. In particular, the exact solutions exhibit inelastic collisions between solitons.

Key words: Coupled Nonlinear Schrödinger system, Bright Soliton, Gauge transformation, Lax pair

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1 Introduction

The investigation of multicomponent solitons, which arise due to the interplay between the second-order dispersion and cubic nonlinearity, has attracted a great deal of attention, starting from the classical paper of Manakov [1], and further enhanced by works on the copropagation of bimodal waves in nonlinear optics [2]-[7]. The dynamics of multicomponent solitons is described by systems of coupled nonlinear Schrödinger (NLS) equations [see, e.g., recent works [8]-[10], and very recent ones [11]-[17] dealing with two-component solitons in spin-orbit-coupled Bose-Einstein condensates (BECs)]. In particular, the concept of energy sharing in the Manakov model [18], or in the modified version of this model [19] governed by coupled NLS equations [20], was a catalyst for looking for new integrable models in nonlinear optics [21], BECs [22,23], metamaterials [24,25], etc. It was found that, in all available integrable systems of two coupled NLS-type equations, the ratio between the self-phase-modulation (SPM) and cross-phase-modulation (XPM) coefficients, which account for the interaction of the components with themselves or with each other, are equal, while physically realistic systems depart from this constraint. Therefore, the quest for new solvable models involving two coupled NLS equations continues. In this context, Park and Shin [26] have developed new forms of integrable NLS-type equations going beyond the framework of the conventional Manakov model, by adding four-wave mixing (FWM) terms to it.

In this paper, we investigate a system of coupled NLS equations including FWM terms and a time-dependent parabolic potential – generally, with an anti-trapping sign, i.e., a potential barrier. An unusual ingredient of the model is that signs in front of the kinetic and gradient terms are opposite in the two equations. Hence, it does not directly apply to known physical systems. Nevertheless, it is interesting as a “non-standard” nonlinear-wave model. We employ the gauge-transformation approach [27] to construct bright-soliton solutions of this system. We conclude that, for a special choice of the trap, bright solitons persist indefinitely long in the system. We verify the analytical results by comparing them to the corresponding numerical simulations, and conclude that the addition of small perturbations, in the form of sudden variation of the trap’s strength, does not destroy the solitons.

2 The model

Waves copropagating in optical media interact through the XPM nonlinearity [7]. Accordingly, the propagation is governed by the Manakov’s model [1] or its generalization [2]-[6]:

$$\begin{aligned}
iq_{1t} + \frac{1}{2}q_{1xx} + 2(g_{11}|q_1|^2 + g_{12}|q_2|^2)q_1 &= 0, \\
iq_{2t} + \frac{1}{2}q_{2xx} + 2(g_{21}|q_1|^2 + g_{22}|q_2|^2)q_2 &= 0,
\end{aligned} \tag{1}$$

where $q_j(x, t)$ ($j = 1, 2$) are envelopes of the field components, g_{11} and g_{22} account for the strengths of the SPM, while g_{12} and g_{21} represent the XPM. It is known that eqs. (1) are integrable if either (i) $g_{11} = g_{12} = g_{21} = g_{22}$ or (ii) $g_{11} = g_{21} = -g_{12} = -g_{22}$. The former choice corresponds to the Manakov model proper [1,28,29] which has been studied in full detail [18,30,31]. The latter choice corresponds to the modified Manakov model [19], in which the soliton dynamics has been explored too.

In addition to the XPM, models of the bimodal light propagation in nonlinear birefringent optical fibers include the FWM terms, $q_1^2 q_2^*$ and $q_2^2 q_1^*$, which account for the coherent nonlinear interaction between two linear polarizations of the electromagnetic waves [2,7]. Taking this into regard, we here address a novel system of coupled NLS-type equations, including the SPM, XPM, and FWM terms with a time-dependent coefficient, and a time-dependent anti-trapping parabolic potential. The equations are written in the notation corresponding to BEC models based on Gross-Pitaevskii (GP) equations [22,23]:

$$\begin{aligned}
iq_{1t} + \frac{1}{2}q_{1xx} + \gamma(t)(|q_1|^2 - 2|q_2|^2)q_1 - \gamma(t)q_2^2 q_1^* + \frac{1}{2}\lambda^2(t)x^2 q_1 &= 0, \\
iq_{2t} + \frac{1}{2}q_{2xx} + \gamma(t)(2|q_1|^2 - |q_2|^2)q_2 + \gamma(t)q_1^2 q_2^* + \frac{1}{2}\lambda^2(t)x^2 q_2 &= 0,
\end{aligned} \tag{2}$$

where $\gamma(t)$ is the strength of the FWM terms, and $\lambda^2(t)$ is the strength of the anti-trapping (expulsive) potential. These potentials occur in various physical contexts, such as the interaction of optical and matter-wave solitons with barriers [32]-[35], and splitting of wave packets in interferometers [36].

Equations (2) can be derived from the Lagrangian,

$$\begin{aligned}
L = & \frac{i}{2} \left(q_1^* \frac{\partial q_1}{\partial t} - q_1 \frac{\partial q_1^*}{\partial t} \right) - \frac{1}{2} \left| \frac{\partial q_1}{\partial x} \right|^2 + \frac{1}{2} \gamma(t) |q_1|^4 + \frac{1}{2} \lambda^2(t) x^2 |q_1|^2 \\
& - 2\gamma(t) |q_1|^2 |q_2|^2 - \frac{1}{2} \gamma(t) \left[q_2^2 (q_1^*)^2 + q_1^2 (q_2^*)^2 \right] - \frac{i}{2} \left[q_2^* \frac{\partial q_2}{\partial t} - q_2 \frac{\partial q_2^*}{\partial t} \right] \\
& + \frac{1}{2} \left| \frac{\partial q_2}{\partial x} \right|^2 + \frac{1}{2} \gamma(t) |q_2|^4 - \frac{1}{2} \lambda^2(t) x^2 |q_2|^2,
\end{aligned}$$

with $*$ standing for the complex conjugate. An obvious peculiarity of the Lagrangian is that it is *sign-indefinite*, as the kinetic and gradient terms, which contain the t - and x -derivatives, respectively, feature opposite signs for

components q_1 and q_2 . For this reason, this system, with “opposite directions” of time in the two subsystems, does not describe currently known physical settings, although it is somewhat similar to the highly idealized model of an optical coupler built of normal and negative-refractive-index cores [37]. Nevertheless, the system seems quite interesting in its own right, as a “non-standard” nonlinear-wave model.

Another noteworthy consequence of the opposite “time directions” in the two subsystems is nonconservation of the usually defined total norm,

$$N = \int_{-\infty}^{+\infty} (|q_1(x)|^2 + |q_2(x)|^2) dx. \quad (3)$$

Indeed, a straightforward corollary of eq. (2) is the following evolution equation for the norm:

$$\frac{dN}{dt} = 4\gamma(t) \int_{-\infty}^{+\infty} \text{Im} \left\{ (q_1^*(x))^2 q_2^2(x) \right\} dx. \quad (4)$$

On the other hand, it is easy to check that the system conserves the *difference* between the norms of the two subsystems:

$$\frac{d}{dt} \int_{-\infty}^{+\infty} (|q_1(x)|^2 - |q_2(x)|^2) dx = 0, \quad (5)$$

which is the manifestation of the conservative character of the system with the “opposite time directions”. In fact, eq.(5) represents the conservation of energy of the dynamical system described by eq.(2). In addition, one can construct several conserved quantities as in [38] consolidating the integrability of eq.(2). Thus, on the contrary to the “normal” systems, where coherent nonlinear coupling leads to exchange of the norm between the subsystems with the conservation of the total norm, here the opposite time directions allow the coherent coupling to generate or absorb the norm.

3 The Lax pair

Equations (2) admits the following Lax-pair representation:

$$\Phi_x + U\Phi = 0, \quad (6)$$

$$\Phi_t + V\Phi = 0, \quad (7)$$

where a three-component Jost function is $\Phi = (\phi_1, \phi_2, \phi_3)^T$, and

$$U = \begin{pmatrix} i\zeta(t) & P(x, t)K(t) & Q(x, t)K(t) \\ -R1(x, t)K(t) & -i\zeta(t) & 0 \\ -R2(x, t)K(t) & 0 & -i\zeta(t) \end{pmatrix}, \quad (8)$$

$$V = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix}, \quad (9)$$

with

$$\begin{aligned} V_{11} &= -i\zeta(t)^2 + i\Omega(t)x\zeta(t) + \frac{i}{2}\gamma(t)A(t)P(x, t)R1(x, t)K(t)^2 \\ &\quad + \frac{i}{2}\gamma(t)A(t)Q(x, t)R1(x, t)K(t)^2 \\ V_{12} &= (\Omega(t)x - \zeta(t))P(x, t)K(t) + \frac{i}{2}(P(x, t)K(t))_x \\ V_{13} &= (\Omega(t)x - \zeta(t))Q(x, t)K(t) + \frac{i}{2}(Q(x, t)K(t))_x \\ V_{21} &= -(\Omega(t)x - \zeta(t))R1(x, t)K(t) + \frac{i}{2}(R1(x, t)K(t))_x \\ V_{22} &= i\zeta(t)^2 - i\Omega(t)x\zeta(t) - \frac{i}{2}\gamma(t)A(t)P(x, t)R1(x, t)K(t)^2 \\ V_{23} &= -\frac{i}{2}Q(x, t)R1(x, t)K(t)^2, \\ V_{31} &= -(\Omega(t)x - \zeta(t))R2(x, t)K(t) + \frac{i}{2}(R2(x, t)K(t))_x \\ V_{32} &= -\frac{i}{2}P(x, t)R2(x, t)K(t)^2 \\ V_{33} &= i\zeta(t)^2 - i\Omega(t)x\zeta(t) - \frac{i}{2}\gamma(t)A(t)Q(x, t)R2(x, t)K(t)^2, \end{aligned}$$

with

$$\begin{aligned} P(x, t) &= e^{(i\phi(x, t))}q_1(x, t) \\ Q(x, t) &= e^{(i\phi(x, t))}q_2(x, t) \\ R1(x, t) &= e^{(-i\phi(x, t))}r_1(x, t) \\ R2(x, t) &= e^{(-i\phi(x, t))}r_2(x, t) \end{aligned}$$

where,

$$\begin{aligned}
r_1(x, t) &= -aq_1(x, t)^* - \frac{bq_1(x, t)q_2(x, t)^*}{2q_2(x, t)} + \frac{d_1b_1q_2(x, t)^2q_1(x, t)^*}{2q_1(x, t)^2} \\
r_2(x, t) &= \frac{b_1q_2(x, t)q_1(x, t)^*}{2q_1(x, t)} - cq_2(x, t)^* + \frac{dbq_1(x, t)^2q_2(x, t)^*}{2q_2(x, t)^2} \\
A(t) &= \frac{1}{\gamma(t)}; K(t) = \frac{1}{\sqrt{A(t)}}; \phi(x, t) = \Omega(t)x^2/2
\end{aligned} \tag{10}$$

where a, b, c, d, b_1 and d_1 are arbitrary constants. One can suitably choose these parameters to obtain eq.(2) as the compatibility condition for the Lax pair defined by Eqs. (6)-(9), $U_t - V_x + [U, V] = 0$, while the spectral parameter $\zeta(t)$ obeys the following equation:

$$\zeta'(t) = \Omega(t)\zeta(t), \tag{11}$$

with

$$\lambda^2(t) = \Omega^2(t) - \Omega'(t), \tag{12}$$

$$\Omega(t) = -\frac{d}{dt} \ln \gamma(t) \tag{13}$$

It should be mentioned that Riccati equation.(12) has already been employed to solve GP-type equations [39]-[42]. In fact, the identification of the Riccati-type equation (12) gives the first signature of complete integrability of eq. (2). Equation (12), which determines the parabolic-potential strength, $\lambda^2(t)$, demonstrates that it is related to the FWM strength, $\gamma(t)$, through the integrability condition, which can be derived by substituting eq. (13) in eq. (12):

$$-\gamma''(t)\gamma(t) + 2(\gamma'(t))^2 - \lambda^2(t)\gamma^2(t) = 0. \tag{14}$$

Thus, the system of coupled GP equations (2) (or coupled NLS equations with a time dependent harmonic trap) is completely integrable for suitable choices of $\lambda(t)$ and $\gamma(t)$, which are consistent with equation (14). For constant $\lambda(t) = c_1$, eq. (14) yields $\gamma(t) = e^{c_1 t}$.

It is worthy to mention that the integrable version of eq. (2) can be transformed, by means of substitution

$$q_{1,2}(x, t) = \frac{1}{\sqrt{\gamma(t)l(t)}} Q_{1,2}(X, T) \exp(-i\Omega(t)x^2/2), \tag{15}$$

with $X \equiv x/l(t)$, $T = T(t)$, $dl/dt = 2\Omega l$, and $dT/dt = 1/l^2$, into a system of coupled perturbed NLS equations with constant coefficients,

$$\begin{aligned}
i\frac{\partial Q_1}{\partial T} + \frac{\partial^2 Q_1}{\partial X^2} + (|Q_1|^2 - 2|Q_2|)Q_1 - Q_2^2 Q_1^* &= i\epsilon(t)Q_1, \\
i\frac{\partial Q_2}{\partial T} + \frac{\partial^2 Q_2}{\partial X^2} + (2|Q_1|^2 - |Q_2|)Q_2 - Q_1^2 Q_2^* &= i\epsilon(t)Q_2.
\end{aligned} \tag{16a}$$

where $\epsilon(t) = (\Omega(t) + \frac{1}{\gamma(t)} \frac{d\gamma(t)}{dt})l^2$. Thus, if we choose $\Omega(t) = -\frac{1}{\gamma(t)} \frac{d\gamma(t)}{dt}$, $\epsilon(t) = 0$ and the above equation reduces to coherently coupled NLS equation investigated recently in Ref. [43] by means of Hirota method.

4 Persistent solitons and collisional dynamics

4.1 Analytical results

To generate bright vector solitons of eq. (2), we now consider the vacuum solution ($q_1^0 = q_2^{(0)} = 0$), so that the corresponding eigenvalue problem becomes

$$\Phi_x^{(0)} = U^{(0)} \Phi^{(0)}, \tag{17}$$

$$\Phi_t^{(0)} = V^{(0)} \Phi^{(0)}, \tag{18}$$

where

$$U^{(0)} = \begin{pmatrix} -i\zeta(t) & 0 & 0 \\ 0 & i\zeta(t) & 0 \\ 0 & 0 & i\zeta(t) \end{pmatrix}, \tag{19}$$

$$V^{(0)} = \begin{pmatrix} -i\zeta(t)^2 + i\zeta(t)\Omega(t)x & 0 & 0 \\ 0 & i\zeta(t)^2 - \zeta(t)i\Omega(t)x & 0 \\ 0 & 0 & i\zeta(t)^2 - i\zeta(t)\Omega(t)x \end{pmatrix}. \tag{20}$$

Solving this vacuum linear eigenvalue problem, one gets

$$\Phi^{(0)} = \begin{pmatrix} e^{-i\zeta(t)x - i \int \zeta(t)^2 dt} & 0 & 0 \\ 0 & e^{i\zeta(t)x + i \int \zeta(t)^2 dt} & 0 \\ 0 & 0 & e^{i\zeta(t)x + i \int \zeta(t)^2 dt} \end{pmatrix}. \tag{21}$$

We now gauge transform the vacuum eigenfunction $\Phi^{(0)}$ by a transformation function $g(x, t)$ to obtain

$$U^{(1)} = gU^{(0)}g^{-1} + g_xg^{-1}, \quad (22)$$

$$V^{(1)} = gV^{(0)}g^{-1} + g_tg^{-1}. \quad (23)$$

We choose transformation function $g(x, t)$ from the solution of the associated Riemann problem such that it is meromorphic in the complex ζ plane, as

$$g(x, t; \zeta) = \left[1 + \frac{\zeta_1 - \zeta_1^*}{\zeta - \zeta_1} P(x, t) \right] \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (24)$$

The inverse of matrix g is given by

$$g^{-1}(x, t; \zeta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \left[1 - \frac{\zeta_1 - \zeta_1^*}{\zeta - \zeta_1^*} P(x, t) \right], \quad (25)$$

where ζ_1 is an arbitrary complex parameter and P is a 3×3 projection matrix ($P^2 = P$) to be determined. The fact that $U^{(1)}$ and $V^{(1)}$ do not develop singularities around $\zeta = \zeta_1$ and $\zeta = \zeta_1^*$ imposes the following constraints on P :

$$P_x = (1 - P)JU^{(0)}(\zeta_1^*)JP - PJU^{(0)}(\zeta_1)J(1 - P), \quad (26)$$

$$P_t = (1 - P)JV^{(0)}(\zeta_1^*)JP - PJV^{(0)}(\zeta_1)J(1 - P), \quad (27)$$

where

$$J = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (28)$$

From the above, it is obvious that one can generate projection matrix $P(x, t)$ using a vacuum eigenfunction, $\Phi^{(0)}(x, t)$ as $P = J \cdot \tilde{P} \cdot J$, where

$$\tilde{P} = \frac{M^{(1)}}{\text{Trace}[M^{(1)}]}, \quad (29)$$

$$M^{(1)} = \Phi^{(0)}(x, t, \zeta_1^*) \cdot \hat{m}^{(1)} \cdot \Phi^{(0)}(x, t, \zeta_1)^{-1}. \quad (30)$$

In the above equation, $\hat{m}^{(1)}$ is a 3×3 arbitrary matrix taking the following form

$$\hat{m}^{(1)} = \begin{pmatrix} e^{2\delta_1}\sqrt{2} & \varepsilon_1^{(1)}e^{2i\chi_1} & \varepsilon_2^{(1)}e^{2i\chi_1} \\ \varepsilon_1^{*(1)}e^{-2i\chi_1} & e^{-2\delta_1}/\sqrt{2} & 0 \\ \varepsilon_2^{*(1)}e^{-2i\chi_1} & 0 & e^{-2\delta_1}/\sqrt{2} \end{pmatrix}, \quad (31)$$

such that the determinant of $M^{(1)}$ becomes zero under condition $|\varepsilon_1^{(1)}|^2 + |\varepsilon_2^{(1)}|^2 = 1$. Thus, choosing $\zeta_1 = \alpha_1 + i\beta_1$ and using eq. (30), matrix $M^{(1)}$ can be explicitly written as

$$M^{(1)} = \begin{pmatrix} e^{-\theta_1}\sqrt{2} & e^{-i\xi_1}\varepsilon_1^{(1)} & e^{-i\xi_1}\varepsilon_2^{(1)} \\ e^{i\xi_1}\varepsilon_1^{*(1)} & e^{\theta_1}/\sqrt{2} & 0 \\ e^{i\xi_1}\varepsilon_2^{*(1)} & 0 & e^{\theta_1}/\sqrt{2} \end{pmatrix}, \quad (32)$$

where

$$\theta_1 = 2x\beta_1(t) - 4 \int (\alpha_1(t)\beta_1(t))dt + 2\delta_1, \quad (33)$$

$$\xi_1 = 2x\alpha_1(t) - 2 \int (\alpha_1(t)^2 - \beta_1(t)^2)dt - 2\chi_1, \quad (34)$$

with $\{\alpha_1(t), \beta_1(t)\} = \{\alpha_{10}, \beta_{10}\} \exp(-\int \Omega(t)dt)$, while δ_1 and χ_1 are arbitrary parameters.

Now, substituting eqs. (24) and (25) in eq. (22), we obtain

$$U^{(1)} = \begin{pmatrix} -i\zeta(t) & U^{(0)} & V^{(0)} \\ -U^{(0)*} & i\zeta(t) & 0 \\ -V^{(0)*} & 0 & i\zeta(t) \end{pmatrix} - 2i(\zeta_1 - \zeta_1^*) \begin{pmatrix} 0 & \tilde{P}_{12} & \tilde{P}_{13} \\ -\tilde{P}_{12} & 0 & 0 \\ -\tilde{P}_{12} & 0 & 0 \end{pmatrix}, \quad (35)$$

and similarly for $V^{(1)}$. Thus, one can write down the one-soliton solution as

$$U^{(1)} = U^{(0)} - 2i(\zeta_1 - \zeta_1^*)\tilde{P}_{12}, \quad (36)$$

$$V^{(1)} = V^{(0)} - 2i(\zeta_1 - \zeta_1^*)\tilde{P}_{13}. \quad (37)$$

Thus, the explicit form of one soliton solution can be written as

$$q_1^{(1)} = 2A_1\varepsilon_1^{(1)}\beta_0\text{sech}(\theta_1)e^{i(-\xi_1+\phi(x,t))}, \quad (38)$$

$$q_2^{(1)} = 2A_2\varepsilon_2^{(1)}\beta_0\text{sech}(\theta_1)e^{i(-\xi_1+\phi(x,t))}, \quad (39)$$

where $\alpha(t), \beta(t)$ are time-dependent scattering lengths and $\varepsilon_{1,2}$ are coupling parameters,

with $\phi(x, t) = \Omega(t)x^2/2$, $A_1 = A_2 = \exp[(1/2)\int \Omega(t)dt]$, while δ_1 and χ_1 are arbitrary parameters, and $\varepsilon_1^{(1)}, \varepsilon_2^{(1)}$ are coupling constants, which are subject to constraint $|\varepsilon_1^{(1)}|^2 + |\varepsilon_2^{(1)}|^2 = 1$.

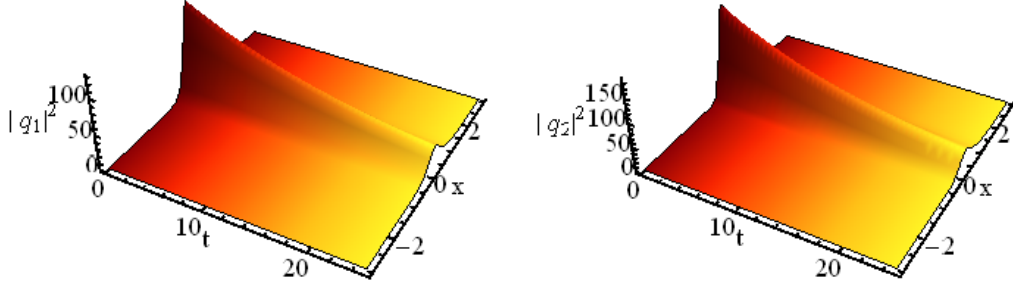


Fig. 1. (Color online) Decay of the soliton solution in the time-independent parabolic potential for $\Omega(t) = -0.02$ (or $\gamma(t) = \exp[0.02t]$), $\varepsilon_1^{(1)} = 0.3$, $\alpha_{10} = 0.1$, $\beta_{10} = 0.3$, $\chi_1 = 0.1$, $\delta_1 = 0.2$.

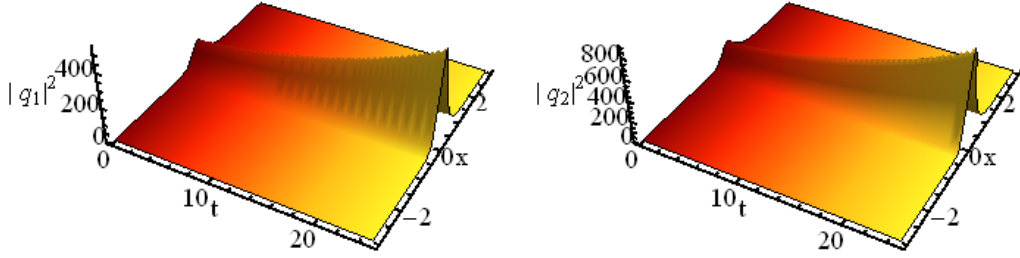


Fig. 2. (Color online) Growth of the soliton solution for $\Omega(t) = 0.02$ (or $\gamma(t) = \exp[-0.02t]$), $\varepsilon_1^{(1)} = 0.3$, $\alpha_{10} = 0.5$, $\beta_{10} = 0.5$, $\chi_1 = 0.5$, $\delta_1 = 0.2$.

Figures 1 and 2 show that, in the case of the time-independent parabolic potential, one observes either decay or growth of the bright solitons, for a suitable choice of the potential's strength ($\Omega(t) = \text{const}$). It should be also mentioned that the growth and decay of solitons is a characteristic feature of variable-coefficient NLS-type equations. For example, the density of the condensates with exponentially varying scattering length in a parabolic trap grows or decays [44] with time, depending on the sign of potential, while the underlying dynamical system is completely integrable and conservative.

To stabilize the solitons, we now introduce a time dependence of the parabolic potential, selecting $\Omega(t)$ as shown in Fig. 3. For this case, the density profile of the solution, shown in Fig. 4, indicates that one can sustain the shape of the bright soliton. Accordingly, we call solutions of the type shown in Fig. 4 “persistent bright solitons”.

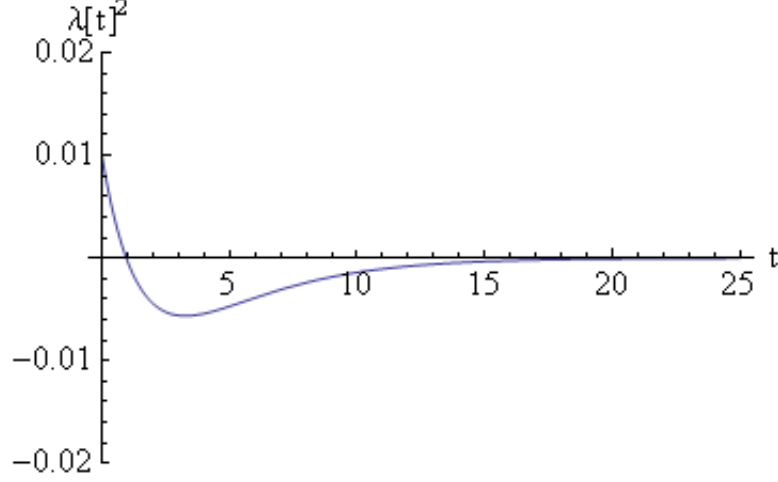


Fig. 3. (Color online) Evolution of the strength of the parabolic potential, $\lambda^2(t)$ (which may be both positive and negative), given by Eq. (14), for $\gamma(t) = \exp \left[(2/3) (1 - e^{-0.3t}) \right]$.

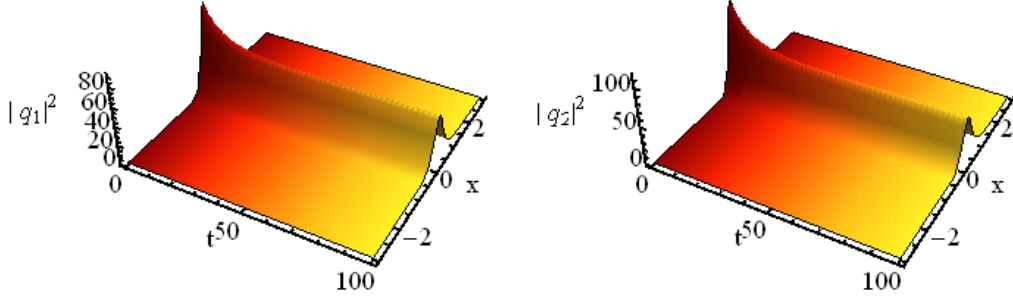


Fig. 4. (Color online) A persistent soliton for the same $\gamma(t)$ as in Fig. 3, and $\varepsilon_1^{(1)} = 0.3$, $\alpha_{10} = 0.2$, $\beta_{10} = 0.5$, $\chi_1 = 0.5$, $\delta_1 = 0.2$.

4.2 Numerical verification

It is possible to confirm the analytical results by numerical solutions of eq. (2), produced by means of the split-step Crank-Nicolson method. In Fig. 5, we have plotted the persistent bright solitons derived analytically as per eqs. (38) and (39) at $t = 10$ and 20 , and the corresponding numerically generated density profiles. Thus, Fig.(5) demonstrates exact matching of the analytical solutions to their numerical counterparts.

Since bright solitons exist due to the special choice of the strength of the parabolic potential as a function of time (see eqs. (12)-(14)), we have also tested the structural stability of the solitons, by suddenly varying the strength of the potential (either increasing or decreasing it by 10%), as shown in Figs. (6) and (7). From figs. (6) and (7), we observe that the addition of a small perturbation does not impact the stability of persistent solitons.

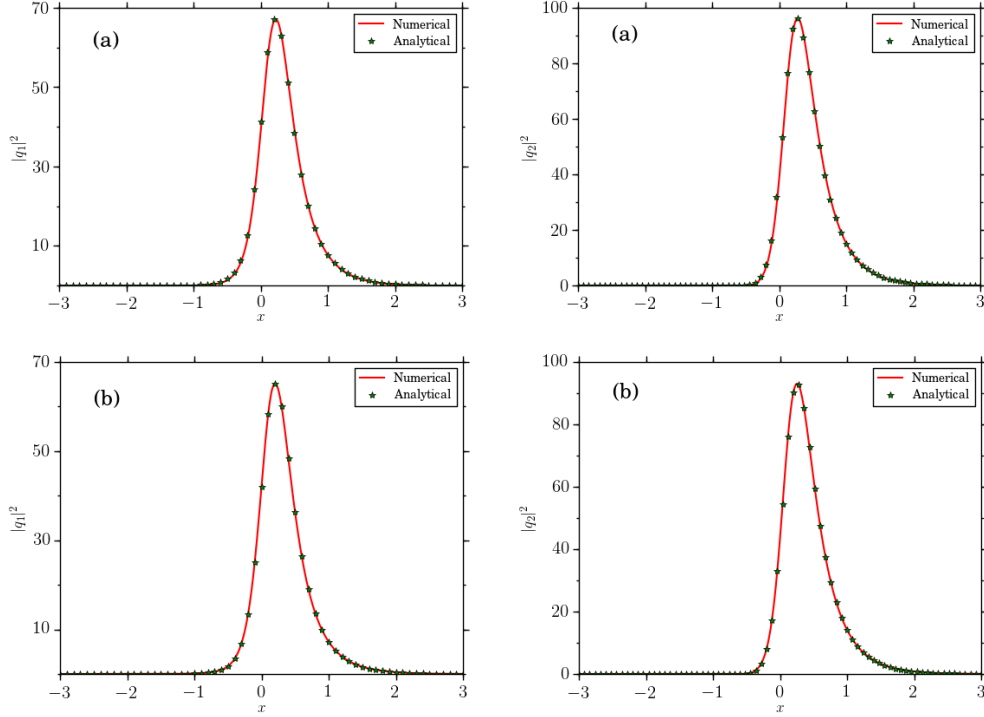


Fig. 5. (Color online) Comparison of analytical and numerical solutions for solitons. (a) The upper left panel: the q_1 component at $t = 10$; the upper right panel: q_2 at $t = 10$. (b) The bottom left panel: q_1 at $t = 20$; the bottom right panel: q_2 at $t = 20$. Parameters are the same as in Fig. (4).

5 Collisional dynamics of bright vector solitons

The gauge-transformation approach can be easily extended to generate multi-soliton solutions [27]. In particular,, the two-soliton solution $q_{1,2}^{(2)}$ for the two modes can be expressed as

$$q_1^{(2)} = 2IA_1/B, \quad q_2^{(2)} = 2IA_2/B, \quad (40)$$

where

$$\begin{aligned} A_1 &= M_{121}M_{222}(\zeta_2 - \zeta_1)(\zeta_1 - \zeta_1^*)(\zeta_2 - \zeta_2^*) + M_{122}M_{221}(\zeta_2 - \zeta_1^*)(\bar{\zeta}_2 - \zeta_1)(\zeta_2 - \zeta_2^*) \\ &\quad + M_{111}M_{122}(\zeta_2 - \zeta_1^*)(\zeta_2^* - \zeta_1^*)(\zeta_2 - \zeta_2^*) + M_{112}M_{121}(\zeta_1 - \zeta_1^*)(\zeta_2^* - \zeta_1)(\zeta_2^* - \zeta_1^*), \\ A_2 &= M_{112}M_{211}(\zeta_2 - \zeta_1)(\zeta_1 - \zeta_1^*)(\zeta_2 - \zeta_2^*) + M_{111}M_{212}(\zeta_2 - \zeta_1^*)(\zeta_2^* - \zeta_1)(\zeta_2 - \zeta_2^*) \\ &\quad + M_{212}M_{221}(\zeta_2 - \zeta_1^*)(\zeta_1 - \zeta_1^*)(\zeta_2 - \zeta_2^*) + M_{211}M_{222}(\zeta_1 - \zeta_1^*)(\zeta_2^* - \zeta_1)(\zeta_2^* - \zeta_1^*), \\ B &= (M_{122}M_{211} + M_{121}M_{212})(\zeta_1 - \zeta_1^*)(\zeta_2 - \zeta_2^*) + (M_{112}M_{221} + M_{111}M_{222})(\zeta_2 - \zeta_1) \\ &\quad (\zeta_2^* - \zeta_1) + (M_{111}M_{112} + M_{221}M_{222})(\zeta_2 - \zeta_1)(\zeta_2^* - \zeta_1^*), \end{aligned}$$

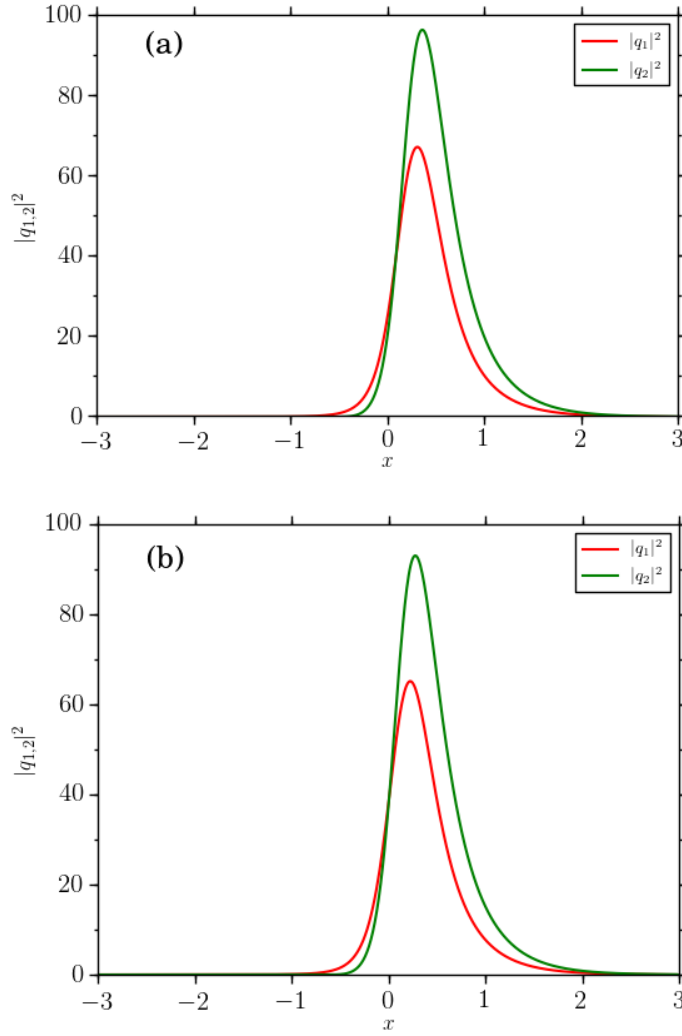


Fig. 6. (Color online) Density profiles of the fields produced by suddenly increasing the potential's strength for, $\gamma(t) = \exp[(20/3)(1 - e^{-0.3t})]$, at (a) $t = 10$, (b) $t = 20$.

$$\begin{aligned}
M_{11j} &= e^{-\theta_j} \sqrt{2}; & M_{12j} &= e^{-i\xi_j} \varepsilon_1^{(j)}; & M_{13j} &= e^{-i\xi_j} \varepsilon_2^{(j)}; \\
M_{21j} &= e^{i\xi_j} \varepsilon_1^{*(j)}; & M_{22j} &= e^{\theta_j} / \sqrt{2}; & M_{23j} &= 0; \\
M_{31j} &= e^{i\xi_j} \varepsilon_2^{*(j)}; & M_{32j} &= 0; & M_{33j} &= e^{\theta_j} / \sqrt{2},
\end{aligned}$$

$$\theta_j = 2x\beta_j(t) - 4 \int (\alpha_j(t)\beta_j(t))dt + 2\delta_j, \quad (41)$$

$$\xi_j = 2x\alpha_j(t) - 2 \int (\alpha_j(t)^2 - \beta_j(t)^2)dt - 2\chi_1, \quad (42)$$

and $j = 1, 2$

In Fig. 8, one can observe inelastic collision of persistent solitons. The collisional dynamics predicted by the analytical solution (the top panel in Fig. 8)

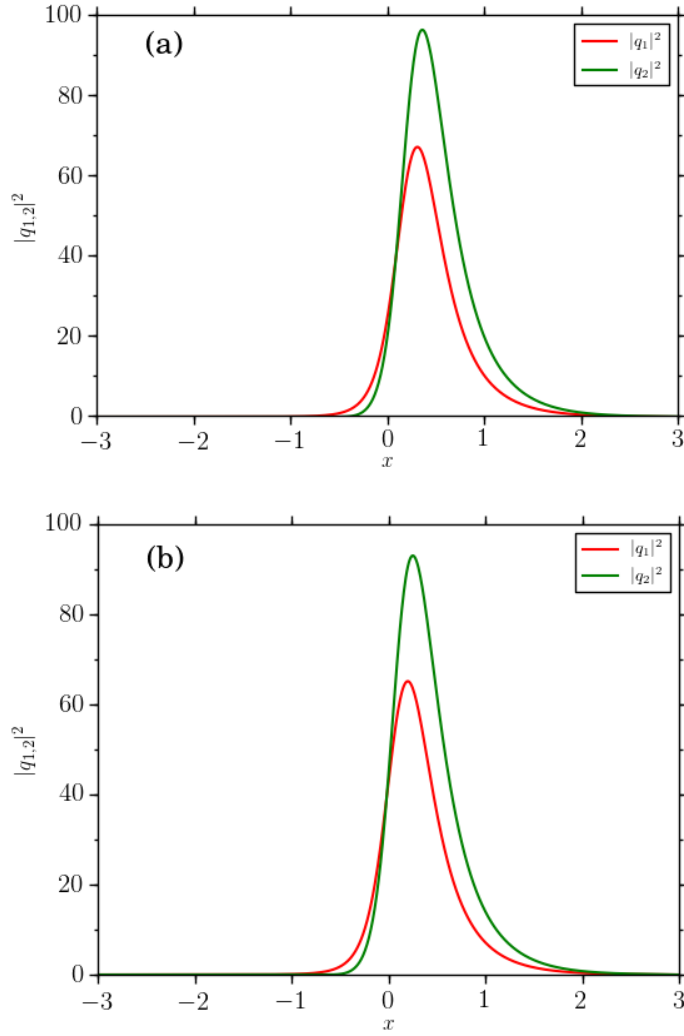


Fig. 7. (Color online) The density profiles of the fields obtained while suddenly decreasing the potential's strength, for $\gamma(t) = \exp[(1/15)(1 - e^{-0.3t})]$ at (a) $t = 10$, (b) $t = 20$.

and its numerical counterpart (the bottom panel in Fig. 8) are identical.

6 Conclusion

The aim of this work is to investigate the dynamics of solitons in the integrable system of coupled NLS equations with “opposite directions” of time in the two subsystems. The system includes the time-dependent nonlinearity coefficient, which must be specifically related to the coefficient in front of the parabolic-potential terms, to secure the integrability. By means of the gauge transformations, we have demonstrated that a special choice of the time dependence of the trap may effectively stabilize bright solitons. We have also

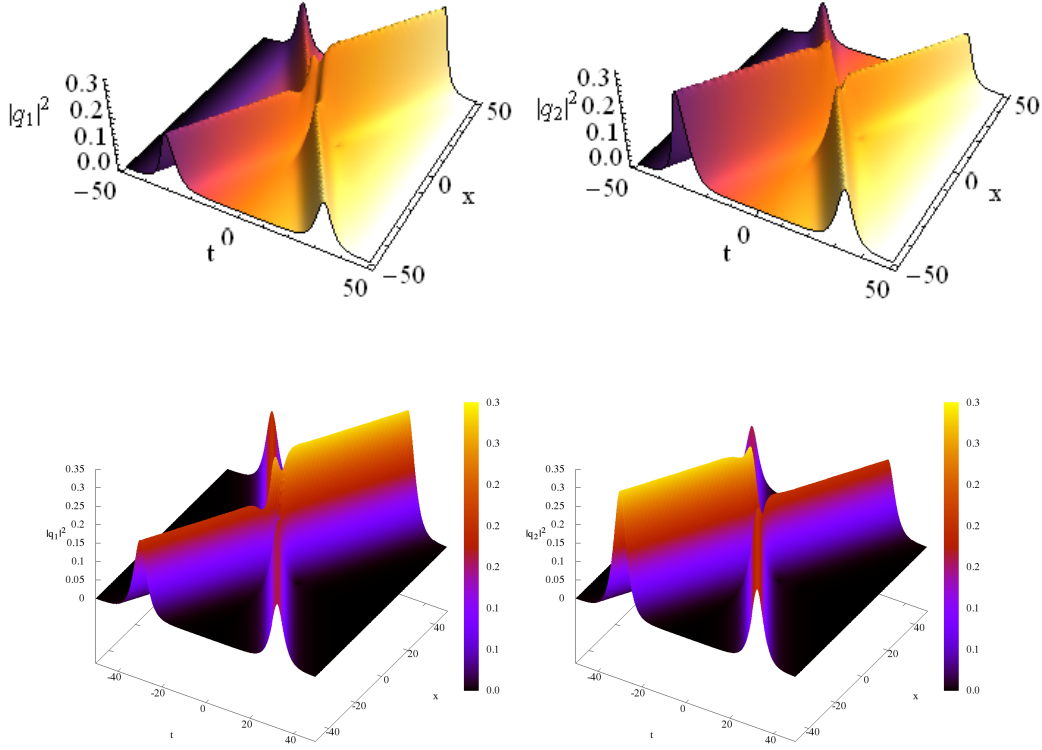


Fig. 8. (Color online) Inelastic collision of solitons for the choice $\gamma(t) = \exp[(2/3)(1 - e^{-0.3t})]$, $\alpha_{10} = 0.1$, $\alpha_{20} = 0.25$, $\beta_{10} = 0.3$, $\beta_{20} = 0.2$, $\delta_1 = 0.1$, $\delta_2 = 0.2$, $\chi_1 = 0.3$, $\chi_2 = 0.4$, $\varepsilon_1^{(1)} = 0.85i$, $\varepsilon_1^{(2)} = 0.5$ such that $|\varepsilon_1^{(j)}|^2 + |\varepsilon_2^{(j)}|^2 = 1$, ($j = 1, 2$). The top and bottom panels show the analytical solution and its numerical counterpart.

observed inelastic collision of persistent solitons for the same choice of trap frequency which is subsequently confirmed by numerical simulations.

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